Calculus

Course Description

Calculus involves the continued study of functions, rates, and accumulation. For Advanced Placement Calculus, a syllabus is supplied by the College Board. For those students electing to take Calculus without the Advanced Placement designation, the following syllabus is suggested. Depending upon the needs of the students and the curricula of other courses offered by districts and/or schools, numerous different syllabi may be written to address these needs by selecting topics in various combinations from the ones listed, as well as others. One sample syllabus is shown below to provide an example of how selected topics may be organized to meet curricular needs of students.

Recommended Prerequisites

This course is appropriate for students who have completed Algebra 1, Algebra 2, Geometry, and Precalculus (or International Baccalaureate Math Methods or International Baccalaureate Math Studies). In particular, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of various functions (e.g., domain and range, odd and even, periodic, symmetry, zeros, intercepts) and be very familiar with the unit circle and the values of trigonometric functions of numbers.

Sample Course Outline

I. Limits of functions.
   A. Understanding the limit process.
      1. Limits that exist.
      2. Limits that fail to exist.
      3. One-sided limits.
      4. Infinite limits.
   B. Calculating limits.
      1. Algebraic.
         a. Cancellation techniques.
         b. Rationalization techniques.
      2. Graphical.
      3. Tabular.

II. Continuity.
   A. Understanding continuity.
      1. Graphical.
      2. Terms of limits.
   B. Properties of continuity.
   C. Intermediate value theorem.
   D. Values of continuity and discontinuity.

III. Asymptotic and unbounded behavior.
   A. Asymptotes in terms of graphical behavior.
   B. Vertical asymptotes.
   C. Limits involving infinity.

D. Horizontal asymptotes.

IV. Derivatives.

A. Concept of the derivative.
   1. Definition as limit of the difference quotient.
   2. Geometric, numeric, and analytic presentations.
   3. Instantaneous rate of change.
   4. Relationship between differentiability and continuity.

B. Finding the derivative of a function.
   1. Constant rule.
   2. Power rule.
   3. Product rule.
   4. Quotient rule.
   5. Chain rule.
   6. All trigonometric functions.
   7. All inverse trigonometric functions.*
   8. Logarithmic.*
   9. Exponential.*
   10. Bases other than e functions.*

C. Derivative at a point.
   1. Slope of a curve at a point.
   2. Tangent line to a curve at a point.
   3. Approximate rate of change from graph and table of values.

D. Relationships of the derivative.
   1. Characteristics of f and f'.
   2. Increasing and decreasing behavior of f and the sign of f'.
   3. Mean value theorem.
   4. Rolle's theorem.

E. Application of derivatives.
   1. Analysis of curves.
   2. Relative extrema of a function.
   3. Optimization.
   4. Modeling rates of change (related rates problems).
   5. Implicit differentiation.
   6. Velocity, speed, and acceleration.
   7. Business and economics (marginal profit, marginal revenue, and marginal cost).

F. Second derivative.
   1. Characteristics of f, f', f?.
   2. Relationship between the concavity of f and the sign of f?.
   3. Points of inflection of a function.

V. Integrals.

A. Techniques of antidifferentiation.
   1. Basic integration rules.
   2. U-substitution (include change of limits of definite integrals).
   3. Antiderivatives for all trigonometric functions.
   4. Antiderivatives for all inverse trigonometric functions.

B. Fundamental theorem of calculus.
    1. Definite integrals.
    2. Representation of an area under a curve.
C. Relationship of integral.*
    1. Specific antiderivative using initial conditions.
    2. Area under the curve.
        a. Using geometry.
        b. Using integration.
    3. Area between two curves.
    4. Numerical integration.
        a. Trapezoidal rule.
        b. Simpson's rule.
    5. Average value.
    6. Separable differential equations.
    7. Growth and decay modeling.
    8. Riemann sums using left, right, and midpoint evaluation points.
    9. Volume.
        a. Disc method.
        b. Washer method.

* These topics could be taught in various sequences.